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## Second Semester B.E./B.Tech. Degree Examination, June/July 2023 Mathematics-II for ME Stream

Time: 3 hrs.

Max. Marks: 100

- Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.  
2. VTU Formula Hand Book is permitted.  
3. M : Marks , L: Bloom's level , C: Course outcomes.*

| Module - 1        |    |   | M | L  | C   |
|-------------------|----|---|---|----|-----|
| Q.1               | a. | Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) dy dx$ .   | 7 | L3 | CO1 |
|                   | b. | Evaluate $\int_0^a \int_x^{\sqrt{a^2-y^2}} y\sqrt{x^2+y^2} dx dy$ by changing into polar.   | 7 | L3 | CO1 |
|                   | c. | Show that $\gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$ .   | 6 | L2 | CO1 |
| <b>OR</b>         |    |   |   |    |     |
| Q.2               | a. | Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy dy dx$ by changing the order of integration.  | 7 | L3 | CO1 |
|                   | b. | Using double integration find the area of the plane in the form of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .  | 7 | L3 | CO1 |
|                   | c. | Using modern mathematical tools, write a program to evaluate :<br>$\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$  | 6 | L3 | CO5 |
| <b>Module - 2</b> |    |   |   |    |     |
| Q.3               | a. | If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ , find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ .   | 7 | L2 | CO2 |
|                   | b. | Find the angle between the surfaces $x^2 + y^2 - z^2 = 4$ and $z = x^2 + y^2 - 13$ at (2, 1, 2).  | 7 | L2 | CO2 |
|                   | c. | Define a irrotational vector. Find the constants a, b, c such that $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$ is irrotational.             | 6 | L2 | CO2 |
| <b>OR</b>         |    |   |   |    |     |
| Q.4               | a. | If $\vec{F} = xy\hat{i} + yz\hat{j} + zx\hat{k}$ , evaluate $\int_c \vec{F} \cdot d\vec{r}$ where c is the curve represented by $x = t, y = t^2, z = t^3, -1 \leq t \leq 1$ . | 7 | L3 | CO2 |
|                   | b. | Using Green's theorem, evaluate $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy$ , where c is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$ .                     | 7 | L3 | CO2 |
|                   | c. | Write the Modern mathematical tool program to find the divergence of the vector field $\vec{F} = x^2yz\hat{i} + y^2zx\hat{j} + z^2xy\hat{k}$ .                                | 6 | L3 | CO5 |

| Module – 3 |       |   |       |       |       |      |      |        |            |       |       |       |       |       |     |    |     |
|------------|-------|---|-------|-------|-------|------|------|--------|------------|-------|-------|-------|-------|-------|-----|----|-----|
| Q.5        | a.    | Form the PDE by eliminating the arbitrary function from $f(x + y + z, x^2 + y^2 + z^2) = 0$   | 7     | L2    | CO3   |      |      |        |            |       |       |       |       |       |     |    |     |
|            | b.    | Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ for which $\frac{\partial z}{\partial y} = -2 \sin y$ , when $x = 0$ and $z = 0$ if $y$ is an odd multiple of $\frac{\pi}{2}$ .  | 7     | L3    | CO13  |      |      |        |            |       |       |       |       |       |     |    |     |
|            | c.    | Derive one dimensional wave equation.   | 6     | L2    | CO3   |      |      |        |            |       |       |       |       |       |     |    |     |
| OR         |       |   |       |       |       |      |      |        |            |       |       |       |       |       |     |    |     |
| Q.6        | a.    | Form the PDE by eliminating the arbitrary constants $a$ and $b$ from $(x - a)^2 + (y - b)^2 + z^2 = r^2$ .  | 7     | L2    | CO3   |      |      |        |            |       |       |       |       |       |     |    |     |
|            | b.    | Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that $x = 0, z = 0$ and $\frac{\partial z}{\partial x} = a \sin y$ .  | 7     | L3    | CO3   |      |      |        |            |       |       |       |       |       |     |    |     |
|            | c.    | Solve $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$ , using Lagrange's multipliers.  | 6     | L3    | CO3   |      |      |        |            |       |       |       |       |       |     |    |     |
| Module – 4 |       |   |       |       |       |      |      |        |            |       |       |       |       |       |     |    |     |
| Q.7        | a.    | Compute the real root of the equation $x \log_{10} x - 1.2 = 0$ by the Regula Falsi method taking four decimal places. Perform three approximation.   | 7     | L3    | CO4   |      |      |        |            |       |       |       |       |       |     |    |     |
|            | b.    | The population of a town is given by the following table:<br><table border="1" style="margin-left: 20px;"> <tr> <td>Year</td> <td>1951</td> <td>1961</td> <td>1971</td> <td>1981</td> <td>1991</td> </tr> <tr> <td>Population</td> <td>19.96</td> <td>39.65</td> <td>58.81</td> <td>72.21</td> <td>94.61</td> </tr> </table> Using Forward and Backward Necoton's interpolation formula, calculate the increase in population between the years 1955 to 1985. | Year  | 1951  | 1961  | 1971 | 1981 | 1991   | Population | 19.96 | 39.65 | 58.81 | 72.21 | 94.61 | 7   | L3 | CO4 |
| Year       | 1951  | 1961  | 1971  | 1981  | 1991  |      |      |        |            |       |       |       |       |       |     |    |     |
| Population | 19.96 | 39.65   | 58.81 | 72.21 | 94.61 |      |      |        |            |       |       |       |       |       |     |    |     |
|            | c.    | Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by taking 7 ordinates using Trapezoidal rule.  | 6     | L3    | CO4   |      |      |        |            |       |       |       |       |       |     |    |     |
| OR         |       |   |       |       |       |      |      |        |            |       |       |       |       |       |     |    |     |
| Q.8        | a.    | Using the Newton Raphson method, find the real root of the equation $x \sin x + \cos x = 0$ , which is nearer to $x = \pi$ , correct to three decimal places.   | 7     | L3    | CO4   |      |      |        |            |       |       |       |       |       |     |    |     |
|            | b.    | Compute the value of $y$ when $x = 4$ using Lagrange's interpolation formula given<br><table border="1" style="margin-left: 20px;"> <tr> <td><math>x</math></td> <td>0</td> <td>2</td> <td>3</td> <td>6</td> </tr> <tr> <td><math>f(x)</math></td> <td>-4</td> <td>2</td> <td>14</td> <td>158</td> </tr> </table>   | $x$   | 0     | 2     | 3    | 6    | $f(x)$ | -4         | 2     | 14    | 158   | 7     | L3    | CO4 |    |     |
| $x$        | 0     | 2   | 3     | 6     |       |      |      |        |            |       |       |       |       |       |     |    |     |
| $f(x)$     | -4    | 2   | 14    | 158   |       |      |      |        |            |       |       |       |       |       |     |    |     |
|            | c.    | Evaluate $\int_0^{\pi/2} \sqrt{\cos \theta} d\theta$ by taking 7 ordinates using Simpson's $\left(\frac{1}{3}\right)^{rd}$ rule.  | 6     | L3    | CO4   |      |      |        |            |       |       |       |       |       |     |    |     |
| Module – 5 |       |   |       |       |       |      |      |        |            |       |       |       |       |       |     |    |     |
| Q.9        | a.    | Use Taylor's series method to find $y(0.1)$ from $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$ .   | 7     | L3    | CO4   |      |      |        |            |       |       |       |       |       |     |    |     |
|            | b.    | Using Runge-Kutta method of order 4, find $y$ at $x = 0.2$ by taking $h = 0.2$ and given $\frac{dy}{dx} = 3x + \frac{y}{2}$ , $y(0) = 1$ .  | 7     | L3    | CO4   |      |      |        |            |       |       |       |       |       |     |    |     |

|           |    |   |   |    |     |
|-----------|----|---|---|----|-----|
|           | c. | Applying Milne's predictor and corrector method, find $y(0.8)$ from $\frac{dy}{dx} = x - y^2$ and given $y(0) = 0$ , $y(0.2) = 0.02$ , $y(0.4) = 0.0795$ , $y(0.6) = 0.1762$ .      | 6 | L3 | CO4 |
| <b>OR</b> |    |   |   |    |     |
| Q.10      | a. | Solve by using modified Euler's method $\frac{dy}{dx} = x - y^2$ , $y(0) = 1$ taking $h = 0.1$ , find $y(0.2)$ .  | 7 | L3 | CO4 |
|           | b. | Using Runge-Kutta method of order 4, find $y$ at $x = 0.1$ , given that $\frac{dy}{dx} = 3e^x + 2y$ , $y(0) = 0$ and $h = 0.1$ .  | 7 | L3 | CO4 |
|           | c. | Using mathematical tools, write a code to find the solution of $\frac{dy}{dx} = 1 + \frac{y}{x}$ at $y(2)$ taking $h = 0.2$ given that $y(1) = 2$ by Runge-Kutta method of order 4. | 6 | L3 | CO4 |

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